Modelling Cosmic Reionization With Resummed Kinetic Field Theory

Final Results

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March 22, 2022

Epoch of Reionization

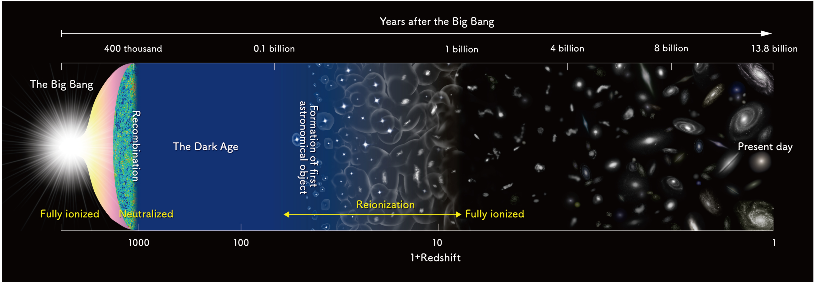

Figure: Evolution of the universe.

Content

Including Baryons in KFT

KFT generating functional

$$Z[\mathbf{J},\mathbf{K}] = e^{i\hat{S}_I} \int \mathrm{d}\Gamma \, \exp\left(i \int \mathrm{d}t \, \boldsymbol{J}(t) \cdot \bar{\boldsymbol{x}}(t) \right) .$$

Including Baryons in KFT

Model baryonic matter as an ideal fluid:

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}(\rho_m \vec{u}) = 0\,,$$
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{\vec{\nabla} P}{\rho_m}\,,$$
$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot (\epsilon \vec{u}) + P \vec{\nabla} \cdot \vec{u} = 0\,.$$

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Close set of equations with equation of state

$$h = \gamma\epsilon = \frac{\gamma}{\gamma - 1} P\,.$$

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$$\frac{\partial h}{\partial t} + (\vec{u} \cdot \vec{\nabla})h + \gamma\, h \vec{\nabla} \cdot \vec{u} = 0\,.$$

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Mesoscopic Particles

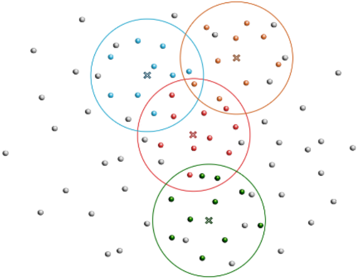

Figure: Idea of mesoscopic particles

Mesoscopic Particles

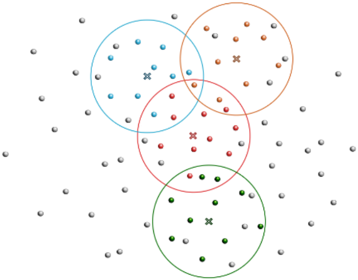

Figure: Idea of mesoscopic particles

- $\vec{q} \hat{=}$ centre of mass of all microscopic particles

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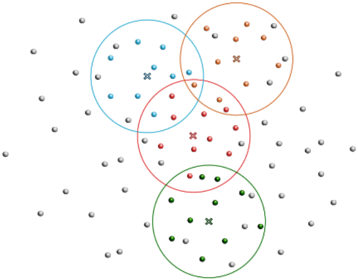

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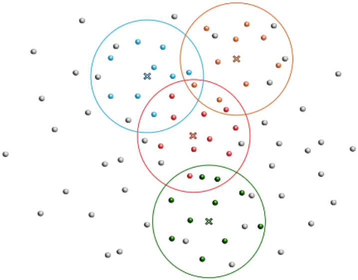

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- $\vec{q} \mathrel{\hat{=}}$ centre of mass of all microscopic particles
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- $\mathcal{H} \mathrel{\hat{=}}$ enthalpy arises from the microscopic random motion

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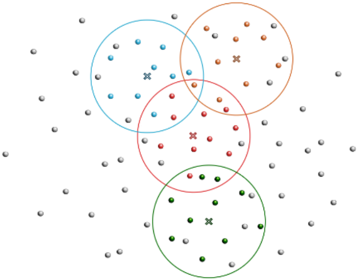

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$\rightarrow$ augment phase space coordinates $(\vec{q}, \vec{p}, \mathcal{H})$.

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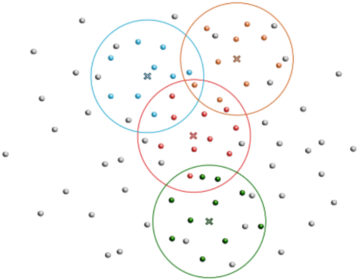

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$\rightarrow$ augment phase space coordinates $(\vec{q}, \vec{p}, \mathcal{H})$.
(The Fourier conjugate variables are $(\vec{k}, \vec{l}, y)$.)

Mesoscopic Particles

Smear out the particle properties with smoothing function w

$$\rho_m(\vec{q},t) = \sum_{j=1}^{N} m w(|\vec{q}-\vec{q}_j(t)|)\,,$$

$$\vec{\pi}(\vec{q},t) = \sum_{j=1}^{N} \vec{p}_j(t) w(|\vec{q}-\vec{q}_j(t)|)\,,$$

$$h(\vec{q},t) = \sum_{j=1}^{N} \mathcal{H}_j(t) w(|\vec{q}-\vec{q}_j(t)|)\,.$$

Smoothing function

$$w(q) = \frac{1}{(2\pi\sigma_w^2)^{3/2}} e^{-q^2/2\sigma_w^2}\,.$$

Equations of Motion for Mesoscopic Particles

Rediscretise the result again, to obtain EOM which can be included in KFT path integral:

$$\dot{\vec{q}}_j = \vec{u}(\vec{q}_j(t), t) = \frac{\vec{p}_j(t)}{m} \, ,$$

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&= -\frac{m}{\rho_m(\vec{q}_j, t)} \frac{\gamma - 1}{\gamma} \sum_{k=1}^{N} \mathcal{H}_k(t) \vec{\nabla}_{q_j} w\left(|\vec{q}_j(t) - \vec{q}_k(t)|\right) \,, \\
\dot{\mathcal{H}}_j(t) &= m \frac{\mathrm{d}}{\mathrm{d}t} \tilde{\mathcal{H}}\left(\vec{q}_j(t), t\right)
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&= -\frac{m}{\rho_m(\vec{q}_j, t)} (\gamma - 1) \sum_{k=1}^{N} \mathcal{H}_k(t) \frac{\vec{p}_k(t) - \vec{p}_j(t)}{m} \vec{\nabla}_{q_j} w\left(|\vec{q}_j(t) - \vec{q}_k(t)|\right) \,.
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Setting up the Action S_I

The interacting part of the action is given by

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$$S_I = \boldsymbol{\mathcal{E}_I} \cdot \boldsymbol{\chi} = \int \mathrm{d}t \, (\vec{\mathcal{E}}_{I,q}(t) \cdot \vec{\chi}_{q_j}(t) + \vec{\mathcal{E}}_{I,p}(t) \cdot \vec{\chi}_{p_j}(t) + \mathcal{E}_{I,\mathcal{H}}(t)\chi_{\mathcal{H}_j})(t)$$

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$$= -\sum_{j=1}^{N} \int \mathrm{d}t \, (\dot{\vec{p}}_j(t) \cdot \vec{\chi}_{p_j}(t) + \dot{\mathcal{H}}_j(t) \chi_{\mathcal{H}_j})(t) \, .$$

Introducing Collective Fields

For computing power spectra reformulate action in terms of collective fields

$$\Phi_f(\vec{x},t) = \sum_{j=1}^{N} \delta_D\left(\vec{q}-\vec{q}_j(t)\right)\delta_D\left(\vec{p}-\vec{p}_j(t)\right)\delta_D\left(\mathcal{H}-\mathcal{H}_j(t)\right)\ ,$$

$$\Phi_{B_\rho}(\vec{x},t) = \sum_{j=1}^{N} \vec{\chi}_{p_j}(t)\cdot\vec{\nabla}_q\delta_D\left(\vec{q}-\vec{q}_j(t)\right)\delta_D\left(\vec{p}-\vec{p}_j(t)\right)\delta_D\left(\mathcal{H}-\mathcal{H}_j(t)\right)\ ,$$

$$\vec{\Phi}_{B_\mathcal{H}}(\vec{x},t) = \sum_{j=1}^{N} \chi_{\mathcal{H}_j}(t)\vec{\nabla}_q\delta_D\left(\vec{q}-\vec{q}_j(t)\right)\delta_D\left(\vec{p}-\vec{p}_j(t)\right)\delta_D\left(\mathcal{H}-\mathcal{H}_j(t)\right)\ .$$

Action S_I in Terms of Collective Fields

In terms of collective fields the action takes the form

$$S_{\psi,I}[\psi] = \int \mathrm{d}1 \int \mathrm{d}2 \left(\Phi_f(-1)\sigma_{fB_\rho}(1,-2)\Phi_{B_\rho}(2) + \Phi_f(-1)\vec{\sigma}_{fB_\mathcal{H}}(1,-2)\cdot\vec{\Phi}_{B_\mathcal{H}}(2)\right) ,$$

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$$\left. + \Phi_f(-1)\vec{\sigma}_{fB_{\mathcal{H}}}(1,-2) \cdot \vec{\Phi}_{B_{\mathcal{H}}}(2) \right) \,,$$

defining the interaction matrix elements

$$\sigma_{fB_\rho}(1,2) = -\frac{m}{\bar{\rho}_m}\frac{\gamma-1}{\gamma} w(k_1) i\partial_{y_1}$$
$$\times (2\pi)^{11}\delta_D(\vec{k}_1+\vec{k}_2)\delta_D(\vec{l}_1)\delta_D(\vec{l}_2)\delta_D(y_1)\delta_D(y_2)\delta_D(t_1-t_2)\,,$$
$$\vec{\sigma}_{fB_{\mathcal{H}}}(1,2) = -\frac{1}{\bar{\rho}_m}(\gamma-1) w(k_1) i\partial_{y_1}(i\vec{\partial}_{l_1} - i\vec{\partial}_{l_2})$$
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defining the interaction matrix elements

$$\begin{aligned}
\sigma_{fB_\rho}(1,2) = &- \frac{m}{\bar{\rho}_m}\frac{\gamma-1}{\gamma} w(k_1) i\partial_{y_1} \\
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\vec{\sigma}_{fB_\mathcal{H}}(1,2) = &- \frac{1}{\bar{\rho}_m}(\gamma-1) w(k_1) i\partial_{y_1}(i\vec{\partial}_{l_1} - i\vec{\partial}_{l_2}) \\
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\end{aligned}$$

To simplify the formalism we introduce the *dressed response field*

$$\Phi_\mathcal{F} = \sigma_{fB_\rho}\cdot\Phi_{B_\rho} + \vec{\sigma}_{fB_\mathcal{H}}\cdot\vec{\Phi}_{B_\mathcal{H}} .$$

Outline

How to Compute Power Spectra With RKFT?

Power spectrum is defined via

$$(2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2)P_\delta(k_1) = \langle\delta(1)\delta(2)\rangle_c =: G_{\delta\delta}(1,2)\,.$$

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The density correlation can be extracted

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Thus, we need to compute G_{ff}:

$$G_{ff}(1,2) = \Delta_{ff}(1,2) + \text{vertex terms}\,.$$

Here, Δ_{ff} is the *statistical propagator*.

Introducing RKFT

Exact reformulation of KFT path integral

$$Z = \int \mathrm{d}\Gamma \int_i \mathcal{D}\psi \; e^{iS_{\psi,0}[\psi] + i\Phi_f \cdot \Phi_{\mathcal{F}}[\psi]}$$

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$$= \int \mathrm{d}\Gamma \int_i \mathcal{D}\psi \int \mathcal{D}f \, e^{iS_{\psi,0}[\psi] + if \cdot \Phi_{\mathcal{F}}[\psi]} \delta_D \left[f - \Phi_f[\psi] \right]$$

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&= \int \mathrm{d}\Gamma \int_i \mathcal{D}\psi \int \mathcal{D}f \, e^{iS_{\psi,0}[\psi]+if\cdot\Phi_{\mathcal{F}}[\psi]} \delta_D\left[f-\Phi_f[\psi]\right] \\
&= \int \mathrm{d}\Gamma \int_i \mathcal{D}\psi \int \mathcal{D}f \int \mathcal{D}\beta \, e^{iS_{\psi,0}[\psi]+if\cdot\Phi_{\mathcal{F}}[\psi]-i\beta\cdot(f-\Phi_f[\psi])}
\end{aligned}
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&= \int \mathrm{d}\Gamma \int_i \mathcal{D}\psi \int \mathcal{D}f \int \mathcal{D}\beta \, e^{iS_{\psi,0}[\psi] + if \cdot \Phi_{\mathcal{F}}[\psi] - i\beta \cdot (f - \Phi_f[\psi])} \\
&= \int \mathcal{D}\phi \, e^{-i\beta \cdot f} Z_{\tilde{\Phi},0}[\tilde{\phi}] \,.
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&= \int \mathcal{D}\phi \, e^{-i\beta \cdot f} Z_{\tilde{\Phi},0}[\tilde{\phi}] \, .
\end{aligned}
$$

We receive the macroscopic action

$$
S_\phi[\phi] \coloneqq -f \cdot \beta - iW_{\tilde{\Phi},0}[\tilde{\phi}] \, ,
$$

with $\phi = (f, \beta)$.

Macroscopic Perturbation Theory

Set up a new perturbative approach to KFT following the standard procedure in quantum and statistical field theory

$$
\begin{aligned}
iS_\phi[\phi] &= iS_\Delta[\phi] + iS_\nu[\phi] \\
&:= -\frac{1}{2}\int \mathrm{d}1 \int \mathrm{d}2\,\phi(-1)\Delta^{-1}(1,2)\phi(-2)
\end{aligned}
$$

$$
+ \sum_{\substack{n_f, n_\beta = 0 \\ n_f + n_\beta \neq 2}} \frac{1}{n_f! n_\beta!} \prod_{u=1}^{n_\beta} \left(\int \mathrm{d}u\,\beta(-u) \right) \prod_{r=1}^{n_f} \left(\int \mathrm{d}r'\,f(-r') \right) \nu_{\beta\ldots\beta f\ldots f}(1, \ldots, n_f')\,.
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$$

Here, Δ^{-1} is the inverse propagator and $\nu_{\beta\ldots\beta f\ldots f}(1, \ldots, n_f')$ the vertex term.

Macroscopic Propagator

Expressions can be found by plugging the functional Taylor series of the macroscopic Schwinger functional $W_{\tilde{\Phi},0}[\tilde{\phi}] = \ln Z_{\tilde{\Phi},0}[\tilde{\phi}]$ into the macroscopic action

$$\Delta(1,2) = \begin{pmatrix} \Delta_{ff} & \Delta_{f\beta} \\ \Delta_{\beta f} & \Delta_{\beta\beta} \end{pmatrix}(1,2) = \begin{pmatrix} \Delta_R \cdot G_{ff}^{(0)} \cdot \Delta_A & -i\Delta_R \\ -i\Delta_A & 0 \end{pmatrix}(1,2)\,,$$

$$\nu_{\beta\ldots\beta f\ldots f}(1,\ldots,n_\beta,1',\ldots,n'_f) = i^{n_f+n_\beta}\; G^{(0)}_{f\ldots f\mathcal{F}\ldots\mathcal{F}}(1,\ldots,n_\beta,1',\ldots,n'_f)\,.$$

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Expressions can be found by plugging the functional Taylor series of the macroscopic Schwinger functional $W_{\tilde{\Phi},0}[\tilde{\phi}] = \ln Z_{\tilde{\Phi},0}[\tilde{\phi}]$ into the macroscopic action

$$\Delta(1,2) = \begin{pmatrix} \Delta_{ff} & \Delta_{f\beta} \\ \Delta_{\beta f} & \Delta_{\beta\beta} \end{pmatrix}(1,2) = \begin{pmatrix} \Delta_R \cdot G^{(0)}_{ff} \cdot \Delta_A & -i\Delta_R \\ -i\Delta_A & 0 \end{pmatrix}(1,2)\,,$$

$$\nu_{\beta\ldots\beta f\ldots f}(1,\ldots,n_\beta,1',\ldots,n'_f) = i^{n_f+n_\beta}\; G^{(0)}_{f\ldots f\mathcal{F}\ldots\mathcal{F}}(1,\ldots,n_\beta,1',\ldots,n'_f)\,.$$

With the definition of the *retarded propagator*

$$\Delta_R(1,2) = \Delta_A(2,1) \coloneqq (\mathcal{I} - iG^{(0)}_{f\mathcal{F}})^{-1}(1,2)\,.$$

Here, $\mathcal{I}$ symbolises the identity two-point function,

$$\mathcal{I}(1,2) \coloneqq (2\pi)^3\delta_D(\vec{k}_1 + \vec{k}_2)\,(2\pi)^3\delta_D(\vec{l}_1 + \vec{l}_2)\,\delta_D(t_1 - t_2)\,.$$

Intermezzo

Expressions for the cumulants or connected correlation functions

$$
\begin{aligned}
&G^{(0)}_{f\ldots B}(1,\ldots,n_f,1',\ldots,n'_B) \\
&\qquad = \frac{\delta}{i\delta H_f(1)}\cdots\frac{\delta}{i\delta H_f(n_f)}\frac{\delta}{i\delta H_B(1')}\cdots\frac{\delta}{i\delta H_B(n'_B)}\,W_{\Phi,0}[H]\bigg|_{H=0}, \\
&G^{(0)}_{f\ldots\mathcal{F}}(1,\ldots,n_f,1',\ldots,n'_{\mathcal{F}}) \\
&\qquad = \frac{\delta}{i\delta H_f(1)}\cdots\frac{\delta}{i\delta H_f(n_f)}\frac{\delta}{i\delta H_{\mathcal{F}}(1')}\cdots\frac{\delta}{i\delta H_{\mathcal{F}}(n'_{\mathcal{F}})}\,W_{\tilde{\Phi},0}[\tilde{H}]\bigg|_{\tilde{H}=0} \\
&\qquad = \prod_{r=1}^{n_{\mathcal{F}}}\left(\int \mathrm{d}\bar{r}\,\sigma_{fB}(r',-\bar{r})\right)\,G^{(0)}_{f\ldots B}(1,\ldots,n_f,\bar{1},\ldots,\bar{n}_{\mathcal{F}})\,.
\end{aligned}
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&G^{(0)}_{f\ldots \mathcal{F}}(1,\ldots,n_f,1',\ldots,n'_{\mathcal{F}}) \\
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\end{aligned}
$$

For actually computing them we need to choose a specific system, i.e. we need equations of motion for $\dot{\vec{q}}, \dot{\vec{p}}, and \dot{\mathcal{H}}$.

Outline

Modelling Reionization

- Photons cannot be directly modelled in KFT
  $\Longrightarrow$ Model the effects of ionising photons

Modelling Reionization

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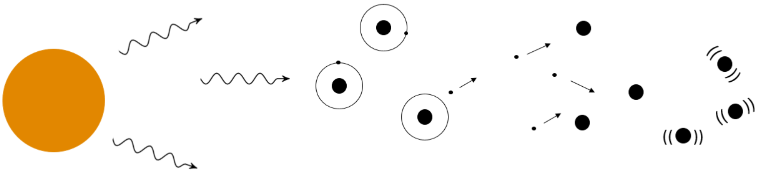
  $\implies$ Model the effects of ionising photons



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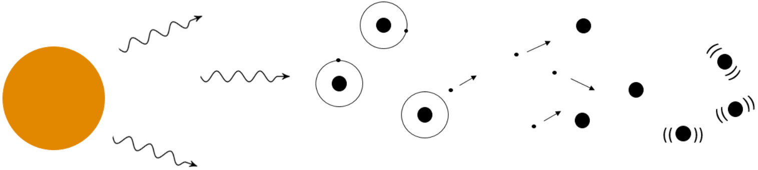

Assumptions:

- only hydrogen
- only ionisation, no recombination
- only ionisation from the ground state

How to Include the Model into KFT?

Add source term to hydrodynamical equations

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}(\rho_m \vec{u}) = 0\,,$$
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{\vec{\nabla} P}{\rho_m}\,,$$
$$\frac{\partial h}{\partial t} + (\vec{u} \cdot \vec{\nabla})h + \gamma\, h \vec{\nabla} \cdot \vec{u} = \mathrm{sources} =: \frac{\mathrm{d}Q_V}{\mathrm{d}t}\,,$$

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Find an expression for the heating rate

$$\frac{\mathrm{d}Q_V}{\mathrm{d}t} = \rho_{H_I}\int_{\nu_L}^{\infty} \mathrm{d}\nu\,(h\nu - h\nu_L)\frac{J_\nu}{h\nu}\sigma_\nu =: \rho_{H_I} S\,.$$

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New Equations of Motion

Again, smooth out the single particle contribution

$$\frac{\mathrm{d}Q_V}{\mathrm{d}t}(\vec{q},t) = \sum_{j=1}^{N} \frac{\mathrm{d}Q_j}{\mathrm{d}t}(T_j, T_\star(t), \rho_j)\, w(|\vec{q} - \vec{q}_j(t)|)$$

$$=: \sum_{j=1}^{N}(1 - x_j)S(t)N\, w(|\vec{q} - \vec{q}_j(t)|)\,.$$

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New equation of motion for the enthalpy

$$\dot{\mathcal{H}}_j(t) = -\frac{m}{\bar{\rho}_m}(\gamma - 1) \sum_{k=1}^{N} \mathcal{H}_k(t) \frac{\vec{p}_k(t) - \vec{p}_j(t)}{m} \vec{\nabla}_{q_j} w\left(|\vec{q}_j(t) - \vec{q}_k(t)|\right)$$

$$+ \frac{m}{\bar{\rho}_m} \gamma \sum_{k=1}^{N} \frac{\mathrm{d}Q_k}{\mathrm{d}t} w\left(|\vec{q}_j(t) - \vec{q}_k(t)|\right)$$

$$=: \quad \dot{\mathcal{H}}_{old,j} + \dot{\mathcal{H}}_{source,j}\,.$$

New Interaction Term

The new equation of motion adds a new term to the interacting part of the action

$$\begin{aligned} S_I &= -\int \mathrm{d}t \, (\dot{\vec{p}}_j \cdot \vec{\chi}_{p_j} + \dot{\mathcal{H}}_j \cdot \chi_{\mathcal{H}_j}) \\ &= -\int \mathrm{d}t \, (\dot{\vec{p}}_j \cdot \vec{\chi}_{p_j} + \dot{\mathcal{H}}_{old,j} \cdot \chi_{\mathcal{H}_j} + \dot{\mathcal{H}}_{source,j} \cdot \chi_{\mathcal{H}_j}) \\ &= S_{I_{old}} + S_{I_{source}} \, . \end{aligned}$$

Set up the Heating Operator

Defining an heating operator

$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k)S(t)N\,.$$

Set up the Heating Operator

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$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k)S(t)N\,.$$

How can we find an ionisation fraction operator?

Set up the Heating Operator

Defining an heating operator

$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k) S(t) N \,.$$

Expression for x obtained from modified Saha equation:

$$\frac{x^2}{1-x} = \frac{(2\pi m_e k_B T)^{3/2}}{h^3 \rho} e^{\frac{h\nu_L}{k_B T}} \times \frac{\int_{\nu_L}^{\infty} \mathrm{d}\nu\, \psi(\nu) \left(\exp\left(\frac{h\nu}{k_B T_\star(t)}\right) - 1\right)^{-1}}{\int_{\nu_L}^{\infty} \mathrm{d}\nu\, \psi(\nu) \exp\left(-\frac{h\nu_L}{k_B T}\right) \left(1 + \left(\exp\left(\frac{h\nu}{k_B T_\star(t)}\right) - 1\right)^{-1}\right)} \,.$$

Set up the Heating Operator

Defining an heating operator

$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k) S(t) N \,.$$

Do a linear Taylor expansion in two variables:

$$x \approx \frac{1}{2} + x_{\rho_0}(\rho - \rho_0) + x_{T_0}(T - T_0)$$

with

$$x_{\rho_0} = \left.\frac{\partial x}{\partial \rho}\right|_{x=\frac{1}{2}} = -\frac{1}{6\rho_0}\,,$$

$$x_{T_0} = \left.\frac{\partial x}{\partial T}\right|_{x=\frac{1}{2}} = \frac{1}{6}\left[\frac{3}{2T_0} + \frac{h\nu_L}{k_B T_0^2} - C(T_0, T_\star)\right]\,,$$

$$C(T_0, T_\star) = \frac{\int \mathrm{d}\nu\, \psi(\nu) \frac{h\nu}{k_B T_0^2} \exp\left(-\frac{h\nu}{k_B T_0}\right)\left(\frac{8\pi h \nu^3}{c^2} + J_\nu(\nu, t)\right)}{\int \mathrm{d}\nu\, \psi(\nu) \exp\left(-\frac{h\nu}{k_B T_0}\right)\left(\frac{8\pi h \nu^3}{c^2} + J_\nu(\nu, t)\right)}\,.$$

Set up the Heating Operator

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Receive for the heating operator

$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k) S(t) N = S(t) N \left[\frac{1}{2} + x_{T_0} T_0 - \frac{2 x_{T_0}}{5 k_B N} \hat{\mathcal{H}}_k \right] \,.$$

Interaction Operator

The interacting part of the action gains an additional term

$$S_{\psi,I}[\psi] = \int \mathrm{d}1 \int \mathrm{d}2\, \Phi_f(-1)\sigma_{fB_\rho}(1,-2)\Phi_{B_\rho}(2)$$
$$+ \Phi_f(-1)\vec{\sigma}_{fB_\mathcal{H}}(1,-2) \cdot \vec{\Phi}_{B_\mathcal{H}}(2) + \Phi_f(-1)\sigma_{fB_{\mathcal{H},source}}(1,-2)\Phi_{source}(1)$$

Here, we defined

$$\Phi_f(1) = \sum_{j=1}^{N} e^{-i\vec{k}_1\vec{q}_j(t_1) - i\vec{l}_1\vec{p}_j(t_1) - iy_1\mathcal{H}_j(t_1)}\,,$$

$$\Phi_{source}(1) = \sum_{j=1}^{N} \chi_{\mathcal{H}_j}(t_1) e^{-i\vec{k}_1\vec{q}_j - i\vec{l}_1\vec{p}_j - iy_1\mathcal{H}_j}\,,$$

$$\sigma_{fB_{\mathcal{H},source}}(1,2) = -\gamma \frac{m}{\bar{\rho}_m} w(k_1)(2\pi)^{11}\delta_D(t_1 - t_2)\delta_D(\vec{k}_1 + \vec{k}_2)$$
$$\times\, \delta_D(\vec{l}_1)\, \delta_D(\vec{l}_2)\, \delta_D(y_1)\, \delta_D(y_2) S(t_1) \left[\frac{1}{2} + x_{T_0} T_0 - \frac{2x_{T_0}}{5k_B N} i\partial_{y_1}\right]\,.$$

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$$\times\,\delta_D(\vec{l}_1)\,\delta_D(\vec{l}_2)\,\delta_D(y_1)\,\delta_D(y_2)S(t_1)\left[\frac{1}{2}+x_{T_0}T_0-\frac{2x_{T_0}}{5k_BN}i\partial_{y_1}\right]\,.$$

Define again

$$\Phi_\mathcal{F} = \sigma_{fB_\rho}\cdot\Phi_{B_\rho} + \vec{\sigma}_{fB_\mathcal{H}}\cdot\vec{\Phi}_{B_\mathcal{H}} + \sigma_{fB_{\mathcal{H},source}}\cdot\Phi_{source}\,.$$

Outline

Ingredients for the Computation of Power Spectra

As shown before, for computing the power spectrum we need to calculate: $G_{ff} \approx \Delta_{ff}$ (on tree-level), $G_{ff}^{(0)} = \langle \Phi_f \Phi_f \rangle_c$ and $G_{f\mathcal{F}_{source}}^{(0)} = \sigma_{fB_{source}} \cdot G_{fB_{source}}^{(0)}$, $G_{fB_{source}}^{(0)} = \langle \Phi_f \Phi_{source} \rangle_c$.

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To compute $G_{fB_{source}}$ we make the following observation:

$$\hat{\Phi}_{source}(r) = \sum_{j=1}^{N} \hat{b}_{source_j}(r) \hat{\Phi}_{f_j}(r) = \sum_{j=1}^{N} \hat{\chi}_{H_j}(t_r) \hat{\Phi}_{f_j}(r) = \sum_{j=1}^{N} \frac{\delta}{i\delta K_{\mathcal{H}_j}(t_r)} \hat{\Phi}_{f_j}(r) \,.$$

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Thus, shift all response operators to the left and first apply the density operators.
Later work out the effect of the response operator an the shifted generating functional

$$\hat{b}_{source_j}(r) Z_0[\mathbf{J} + \mathbf{L}, \mathbf{K}] \bigg|_{\mathbf{J}=0=\mathbf{K}} = \sum_{u \in I_j} y_u g_{\mathcal{H}\mathcal{H}}(t_u, t_r) Z_0[\mathbf{L}, 0] \,.$$

Ingredients for the Computation of Power Spectra

Propagators:

$$g_{qq}(t,t') = g_{pp}(t,t') = g_{\mathcal{H}\mathcal{H}}(t,t') = \Theta(t-t')\,,$$
$$g_{qp}(t,t') = \frac{t-t'}{m}\,,\ g_{pq}(t,t') = 0\,.$$

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Required cumulants:

$$\begin{aligned}
G_{ff}^{(0)}(1,2) &= (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2)\bar{\rho}^2 P_\delta^{(i)}(k_1)e^{i(y_1+y_2)\mathcal{H}_i}\,,\\
G_{f\mathcal{F}}^{(0)}(1,2) &= (2\pi)^7\delta_D(\vec{k}_1+\vec{k}_2)i\bar{\rho}\\
&\times\Big[\Big(v_G(k_1)+v_{P_p}^b(k_1,t_1)i\partial_{y_2}\Big)\Big(k_1^2 g_{qp}(t_1,t_2)+\vec{k}_1\cdot\vec{l}_1\Theta(t_1-t_2)\Big)\\
&+ v_{P_\mathcal{H}}^b(k_1,t_1)y_1\vec{k}_1\cdot i\vec{\partial}_{l_2}i\partial_{y_2}\Theta(t_1-t_2)\\
&-\gamma\frac{m}{\bar{\rho}_m}w(k_2)S(t_2)N\left[\frac{1}{2}+x_{T_0}T_0-\frac{2x_{T_0}}{5k_BN}i\partial_{y_2}\right]y_1\Theta(t_1-t_2)\Big]\\
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G_{f\mathcal{F}}^{(0)}(1,2) &= (2\pi)^7\delta_D(\vec{k}_1+\vec{k}_2)i\bar{\rho}\\
&\times\Big[\Big(v_G(k_1)+v_{P_p}^b(k_1,t_1)i\partial_{y_2}\Big)\Big(k_1^2 g_{qp}(t_1,t_2)+\vec{k}_1\cdot\vec{l}_1\Theta(t_1-t_2)\Big)\\
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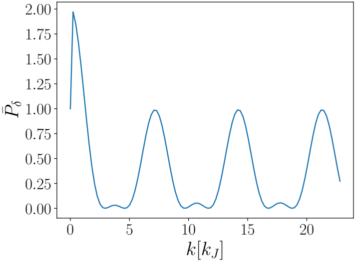
&-\gamma\frac{m}{\bar{\rho}_m}w(k_2)S(t_2)N\left[\frac{1}{2}+x_{T_0}T_0-\frac{2x_{T_0}}{5k_BN}i\partial_{y_2}\right]y_1\Theta(t_1-t_2)\Big]\\
&\times\delta_D(\vec{l}_2)\,\delta_D(y_2)e^{-iy_1\mathcal{H}^{(i)}}\,.
\end{aligned}$$

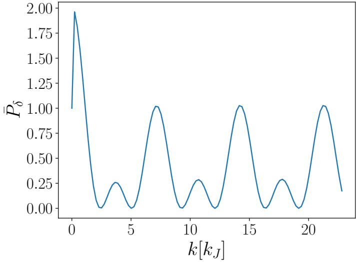
Here, we defined the potentials

$$v_{P_p}^b(k) = \frac{m}{\bar{\rho}_m^b}\frac{\gamma-1}{\gamma}w(k)\,,\, v_{P_\mathcal{H}}^b(k) = \frac{1}{\bar{\rho}_m^b}(\gamma-1)w(k)\,,\, v_G^{\alpha\gamma} = -\frac{4\pi G m^\alpha m^\gamma}{k^2}\,.$$

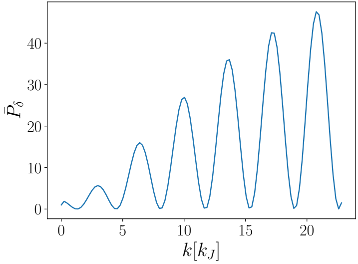
Results for the Matter PS (Constant Part) $S = 3.5 \cdot 10^{-19+s}\ \frac{\text{erg}}{\text{s}}$



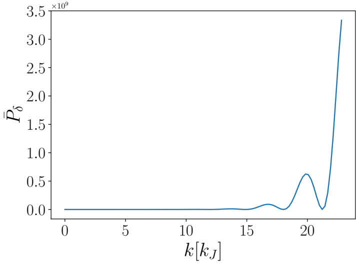
(a) s = 18



(b) s = 19

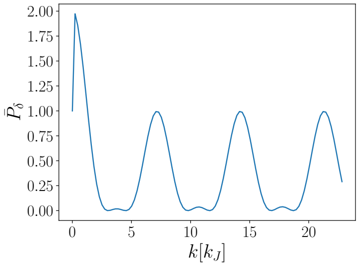


(c) s = 20

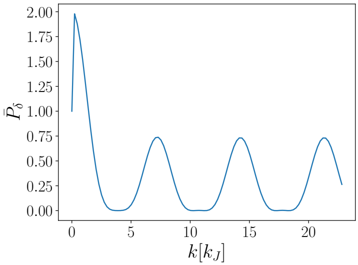


(d) s = 21

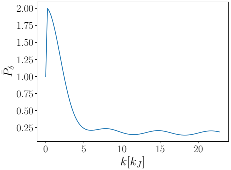
Results for the Matter Power Spectrum (Second Part)



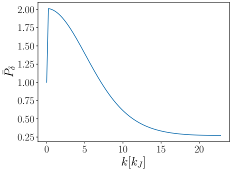
(e) s = 6



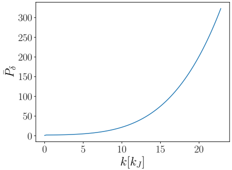
(f) s = 8



(g) s = 9



(h) s = 10



(i) s = 12

Outline

Analytic Toy Model

Start with the hydrodynamical equations again

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}(\rho_m \vec{u}) = 0\,,$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\frac{(\gamma-1)\vec{\nabla} h}{\gamma\rho}\,,$$

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h + \gamma\, h\vec{\nabla}\cdot\vec{u} = (1-x)\,\rho\, S(t) N\,.$$

Adopting the ansatz

$$\rho = \rho_0(1+\delta)\,,\quad \vec{u} = \vec{v}_0 + a\vec{v}\,,\quad P = P_0 + \delta P\,,\quad \Phi = \Phi_0 + \phi\,,\quad h = h_0 + \hbar\,.$$

Analytic Toy Model - Enthalpy Equation

Adopting the ansatz

$$\rho = \rho_0(1+\delta)\,, \quad \vec{u} = \vec{v}_0 + a\vec{v}\,, \quad P = P_0 + \delta P\,, \quad \Phi = \Phi_0 + \phi\,, \quad h = h_0 + \hslash\,,$$

the enthalpy equation

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h + \gamma\, h\vec{\nabla}\cdot\vec{u} = (1-x)\,\rho\, S(t)N$$

Analytic Toy Model - Enthalpy Equation

Adopting the ansatz

$$\rho = \rho_0(1+\delta)\,, \quad \vec{u} = \vec{v}_0 + a\vec{v}\,, \quad P = P_0 + \delta P\,, \quad \Phi = \Phi_0 + \phi\,, \quad h = h_0 + \hbar\,,$$

the enthalpy equation

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h + \gamma\, h\vec{\nabla}\cdot\vec{u} = (1-x)\,\rho\, S(t)N$$

changes to (+ linearised)

$$\dot{h}_0 + \dot{\hbar} + \vec{v}_0\vec{\nabla}\hbar + \gamma\hbar\vec{\nabla}\cdot\vec{v}_0 + \gamma h_0\vec{\nabla}\cdot(\vec{v}_0+\vec{v})$$
$$= S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0(1+\delta) - \frac{2x_{T_0}}{5k_B N}(h_0+\hbar)\right]\,.$$

Analytic Toy Model - Enthalpy Equation

Adopting the ansatz

$$\rho = \rho_0(1+\delta)\,, \quad \vec{u} = \vec{v}_0 + a\vec{v}\,, \quad P = P_0 + \delta P\,, \quad \Phi = \Phi_0 + \phi\,, \quad h = h_0 + \hbar\,,$$

the enthalpy equation

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h + \gamma\, h\vec{\nabla}\cdot\vec{u} = (1-x)\,\rho\, S(t)N$$

changes to (+ linearised)

$$\begin{aligned} &\dot{h}_0 + \dot{\hbar} + \vec{v}_0\vec{\nabla}\hbar + \gamma\hbar\vec{\nabla}\cdot\vec{v}_0 + \gamma h_0\vec{\nabla}\cdot(\vec{v}_0+\vec{v}) \\ &\quad = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0(1+\delta) - \frac{2x_{T_0}}{5k_BN}(h_0+\hbar)\right]\,. \end{aligned}$$

Subtracting the background

$$\dot{h}_0 + \gamma h_0\vec{\nabla}\cdot\vec{v}_0 = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0 - \frac{2x_{T_0}}{5k_BN}h_0\right]$$

Analytic Toy Model - Enthalpy Equation

Adopting the ansatz

$$\rho = \rho_0(1+\delta)\,, \quad \vec{u} = \vec{v}_0 + a\vec{v}\,, \quad P = P_0 + \delta P\,, \quad \Phi = \Phi_0 + \phi\,, \quad h = h_0 + \hbar\,,$$

the enthalpy equation

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h + \gamma\, h\vec{\nabla}\cdot\vec{u} = (1-x)\,\rho\, S(t)N$$

changes to (+ linearised)

$$\dot{h}_0 + \dot{\hbar} + \vec{v}_0\vec{\nabla}\hbar + \gamma\hbar\vec{\nabla}\cdot\vec{v}_0 + \gamma h_0\vec{\nabla}\cdot(\vec{v}_0+\vec{v}) \\ = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0(1+\delta) - \frac{2x_{T_0}}{5k_BN}(h_0+\hbar)\right]\,.$$

Subtracting the background

$$\dot{h}_0 + \gamma h_0\vec{\nabla}\cdot\vec{v}_0 = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0 - \frac{2x_{T_0}}{5k_BN}h_0\right]$$

leads to

$$\dot{\hbar} + \dot{a}\vec{x}\vec{\nabla}\hbar + \gamma h_0\vec{\nabla}\cdot\vec{v} + 3\gamma\hbar\dot{a} = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta - \frac{2x_{T_0}}{5k_BN}\hbar\right]\,.$$

Analytic Toy Model - Enthalpy Equation

Transforming the derivatives to comoving coordinates according to

$$\vec{\nabla} \to a^{-1}\vec{\nabla}\,, \qquad \partial_t \to \partial_t - H\vec{x}\cdot\vec{\nabla}$$

and using the continuity equation

Analytic Toy Model - Enthalpy Equation

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$$\vec{\nabla} \to a^{-1}\vec{\nabla}\,, \qquad \partial_t \to \partial_t - H\vec{x}\cdot\vec{\nabla}$$

and using the continuity equation yields

$$\dot{\hbar} - \gamma\frac{h_0}{a}\delta + 3\gamma\hbar\dot{a} = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta - \frac{2x_{T_0}}{5k_BN}\hbar\right]\,.$$

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For the static case we can set $\dot{a} = 0$ and $a = 1$

Analytic Toy Model - Enthalpy Equation

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For the static case we can set $\dot{a} = 0$ and $a = 1$, thus

$$\dot{\hbar} = \gamma h_0\dot{\delta} + S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta - \frac{2x_{T_0}}{5k_BN}\hbar\right]\,,$$

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$$\dot{\hbar} = \gamma h_0\dot{\delta} + S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta - \frac{2x_{T_0}}{5k_BN}\hbar\right]\,,$$

$$\ddot{\delta} = 4\pi Gm\rho_0\delta + \frac{\gamma-1}{\gamma\rho_0 m}\vec{\nabla}^2\hbar\,.$$

Analytic Toy Model - Enthalpy Equation

Transforming the derivatives to comoving coordinates according to

$$\vec{\nabla} \to a^{-1}\vec{\nabla}\,, \qquad \partial_t \to \partial_t - H\vec{x}\cdot\vec{\nabla}$$

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$$\dot{\hbar} - \gamma\frac{h_0}{a}\delta + 3\gamma\hbar\dot{a} = S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta - \frac{2x_{T_0}}{5k_BN}\hbar\right]\,.$$

For the static case we can set $\dot{a} = 0$ and $a = 1$

$$\ddot{\delta}_k = 4\pi G m\rho_0\delta_k - \frac{\gamma - 1}{\gamma\rho_0 m}k^2\hbar\,,$$

$$\dot{\hbar}_k = \gamma h_0\dot{\delta}_k + S(t)N\left[\left(\frac{1}{2} + x_{T_0}T_0\right)\rho_0\delta_k - \frac{2x_{T_0}}{5k_BN}\hbar_k\right]\,.$$

Solving the Toy Model

Transform this set of equations

$$\ddot{\delta}_k = 4\pi G m \rho_0 \delta_k - \frac{\gamma - 1}{\gamma \rho_0 m} k^2 \hbar \,,$$

$$\dot{\hbar}_k = \gamma h_0 \dot{\delta}_k + S(t) N \left[\left(\frac{1}{2} + x_{T_0} T_0 \right) \rho_0 \delta_k - \frac{2 x_{T_0}}{5 k_B N} \hbar_k \right]$$

into a system of coupled differential equations of first order

$$\dot{y}_0 = y_1 \,,$$

$$\dot{y}_1 = 4\pi G m \rho_0 y_0 - \frac{\gamma - 1}{\gamma \rho_0 m} k^2 y_2 =: a y_0 - b y_2 \,,$$

$$\dot{y}_2 = \gamma h_0 y_1 + S(t) N \left(\frac{1}{2} + x_{T_0} T_0 \right) \rho_0 y_0 - S(t) \frac{2 x_{T_0}}{5 k_B} y_2 =: d y_1 + c y_0 - f y_2 \,.$$

Solving the Toy Model

$$\dot{y}_0 = y_1\,,$$

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Let us collect these equations into a matrix equation

$$\dot{\vec{y}} = M \cdot \vec{y}\,,$$

with the following identifications

$$\vec{y} = \begin{pmatrix} \delta \\ \dot{\delta} \\ \hbar \end{pmatrix}\,, \qquad M = \begin{pmatrix} 0 & 1 & 0 \\ a & 0 & -b \\ c & d & -f \end{pmatrix}\,.$$

For $c = 0 = f$, the equation is solved by

$$\vec{y}(t) = e^{Mt} \vec{C}\,.$$

The Solution of the Toy Model

The solution reads

$$\vec{y}(t) = e^{Mt}\vec{C}\,.$$

with

$$e^{Mt} = \begin{pmatrix} \frac{-bd}{\xi}\cosh(\sqrt{\xi}t) - \frac{bd}{\xi} & \frac{1}{\sqrt{\xi}}\sinh(\sqrt{\xi}t) & \frac{-b}{\xi}\cosh(\sqrt{\xi}t) - \frac{b}{\xi} \\ \frac{a}{\sqrt{\xi}}\sinh(\sqrt{\xi}t) & \cosh(\sqrt{\xi}t) & \frac{-b}{\sqrt{\xi}}\sinh(\sqrt{\xi}t) \\ \frac{ad}{\xi}\cosh(\sqrt{\xi}t) - \frac{ad}{\xi} & \frac{d}{\sqrt{\xi}}\sinh(\sqrt{\xi}t) & \frac{-bd}{\xi}\cosh(\sqrt{\xi}t) + \frac{a}{\xi} \end{pmatrix}.$$

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Interpretation:

The Solution of the Toy Model

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Interpretation:

- two regimes:
  $\xi = a - bd = 0$
  $k_J^2 = \frac{4\pi G \rho_0^2 m^2}{(\gamma-1)h_0}$.

The Solution of the Toy Model

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- two regimes:
  $\xi = a - bd = 0$
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- shift along y-axis

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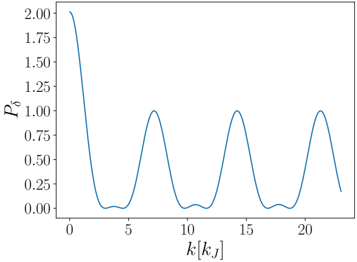
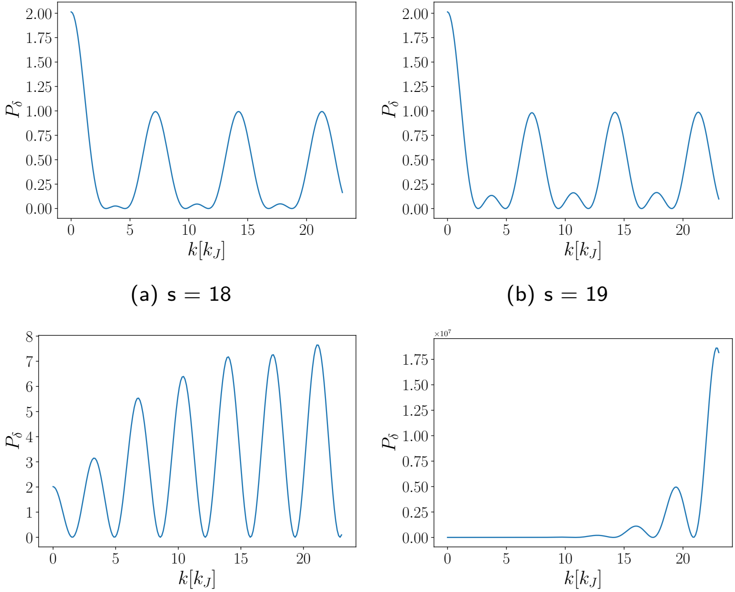


Figure: density contrast power spectrum

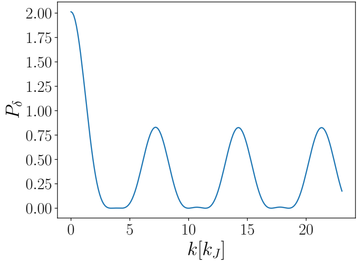
Including Reionization (Constant Part)



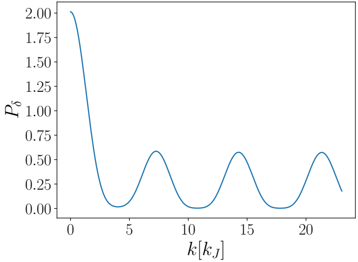
(a) s = 18 (b) s = 19

(c) s = 20 (d) s = 21

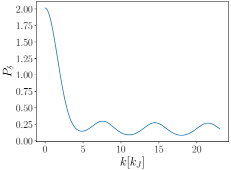
Including Reionization (Second Part)



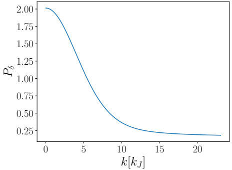
(e) s = 8



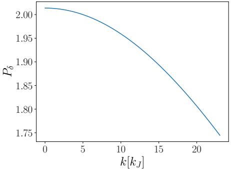
(f) s = 8.5



(g) s = 9



(h) s = 10



(i) s = 12

Structure Growth on Small Scales

The system of equations

$$\dot{y}_0 = y_1 \,,$$

$$\dot{y}_1 = 4\pi G m \rho_0 y_0 - \frac{\gamma - 1}{\gamma \rho_0 m} k^2 y_2 =: a y_0 - b y_2 \,,$$

$$\dot{y}_2 = \gamma h_0 y_1 + S(t) N \left(\frac{1}{2} + x_{T_0} T_0 \right) \rho_0 y_0 - S(t) \frac{2 x_{T_0}}{5 k_B} y_2 =: d y_1 + c y_0 - f y_2 \,.$$

Approximations:

- for small scales/large k: $a = 0$.

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Approximations:

- for small scales/large k: $a = 0$.
- for large heating: $d = 0$.

Structure Growth on Small Scales

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Approximations:

- for small scales/large k: $a = 0$.
- for large heating: $d = 0$.
- for simplicity: $f = 0$.

Structure Growth on Small Scales

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With that the equations simplify to

$$\ddot{\delta} = -b\hbar\,, \qquad \dot{\hbar} = c\delta\,.$$

Structure Growth on Small Scales

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Combine both equations

$$\dddot{\delta} = -bc\delta\,.$$

Structure Growth on Small Scales

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$$\ddot{\delta} = -b\hbar\,, \qquad \dot{\hbar} = c\delta\,.$$

Combine both equations

$$\dddot{\delta} = -bc\delta\,.$$

Solved by

$$\delta(t) = e^{(-bc)^{1/3} t} = e^{(bc)^{1/3} \frac{t}{2}} e^{i (bc)^{1/3} \sin(\frac{\pi}{3}) t}\,.$$

Outline

Coupling of Dark Matter and Baryons

- So far we considered a purely baryonic system.
- For a cosmologically more meaningful model we need to couple dark and baryonic matter.
- Assign some substructure to the collective fields Φ_f^α and $\Phi_{\mathcal{F}}^\alpha$, the propagators, and vertices, with α labelling the particle species. E.g.:

$$\Phi_{\mathcal{F}}^\alpha = \sum_{\gamma=b,d} \left[\sigma_{fB_\rho}^{\alpha\gamma} \cdot \Phi_{B_\rho}^\gamma + \vec{\sigma}_{fB_\mathcal{H}}^{\alpha\gamma} \cdot \vec{\Phi}_{B_\mathcal{H}}^\gamma + \sigma_{fB_{\mathcal{H},source}}^{\alpha\gamma} \cdot \Phi_{source}^\gamma \right]$$

with

$$\sigma_{fB_\rho}^{\alpha\gamma}(1,2) = -\left(v_G^{\alpha\gamma}(k_1, t_1) + \delta_{\alpha b}\delta_{\gamma b} v_{P_p}^b(k_1, t_1) i\partial_{y_1} \right) \times (2\pi)^{11} \delta_D(\vec{k}_1 + \vec{k}_2)\delta_D(\vec{l}_1)\delta_D(\vec{l}_2)\delta_D(y_1)\delta_D(y_2)\delta_D(t_1 - t_2)\,.$$

Adjustments for the Expanding Spacetime

Again, start with the hydrodynamical equations

$$\frac{\partial \rho_{m,c}}{\partial t} - H(\vec{x} \cdot \vec{\nabla})\rho_{m,c} + a^{-1}\vec{\nabla}(\rho_{m,c}\vec{u}) = 0\,,$$

$$\frac{\partial \vec{u}}{\partial t} - H(\vec{x} \cdot \vec{\nabla})\vec{u} + a^{-1}(\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{\gamma - 1}{\gamma} a^{-1} \frac{\vec{\nabla} h_c}{\rho_{m,c}}\,,$$

$$\frac{\partial h_c}{\partial t} - H(\vec{x} \cdot \vec{\nabla})h_c + a^{-1}(\vec{u} \cdot \vec{\nabla})h_c + \gamma\, h_c a^{-1}\vec{\nabla} \cdot \vec{u} = \frac{m\gamma}{\rho_{m,c}} \frac{\mathrm{d}Q_V}{\mathrm{d}t}\,.$$

Enthalpy Equation in Comoving Coordinates

$$\dot{\mathcal{H}}_j(t) = -\frac{m}{\bar{\rho}_m}\frac{\gamma - 1}{a}\sum_{k=1}^{N}\mathcal{H}_k(t)(\vec{u}_k - \vec{u}_j)\vec{\nabla}_{q_j} w\left(|\vec{q}_j(t) - \vec{q}_k(t)|\right)$$

$$+\frac{m\gamma}{\bar{\rho}_m}\sum_{k=1}^{N=1}\frac{\mathrm{d}Q_k}{\mathrm{d}t}(\mathcal{H}_k)w(|\vec{q}_j - \vec{q}_k|)$$

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Plug in the comoving velocity

$$\vec{u} = \dot{\vec{r}} = \dot{a}\vec{x} + a\dot{\vec{x}}\,,$$

and obtain

$$\dot{\mathcal{H}}_j(t) = -\frac{\gamma-1}{\bar{\rho}}\sum_{k=1}^{N}\mathcal{H}_k(t)\left[H(\vec{x}_k-\vec{x}_j)+(\dot{\vec{x}}_k-\dot{\vec{x}}_j)\right]\vec{\nabla}_{q_j}w\left(|\vec{q}_j(t)-\vec{q}_k(t)|\right) + \frac{m\gamma}{\bar{\rho}_m}\sum_{k=1}^{N=1}\frac{\mathrm{d}Q_k}{\mathrm{d}t}(\mathcal{H}_k)w(|\vec{q}_j-\vec{q}_k|)\,.$$

Heating Term in Comoving Coordinates

The heating operator is given as

$$\frac{\mathrm{d}\hat{Q}_k}{\mathrm{d}t} = (1 - \hat{x}_k)S(t)N\,.$$

Assumptions:

- emission, photoionisation, and the heating in same frame of reference $\implies S \to S$.

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Introduce new time coordinate $\eta = ln(\frac{a}{a_i})$

$$\mathcal{H}_j' = -\frac{\gamma - 1}{\bar{\rho}H} \sum_{k=1}^{N} \mathcal{H}_k \left[H(\vec{x}_k - \vec{x}_j) + H(\vec{x}_k' - \vec{x}_j')\right] \vec{\nabla}_{q_j} w\left(|\vec{q}_j - \vec{q}_k|\right)$$
$$+ \frac{m\gamma}{\bar{\rho}_m H} \sum_{k=1}^{N=1} \frac{\mathrm{d}Q_k}{\mathrm{d}t}(\mathcal{H}_k) w(|\vec{q}_j - \vec{q}_k|)\,.$$

Propagators and Potentials for an Expanding Spacetime

It turns out that the whole formalism stays the same, we just have to consider the new propagators

$$
\begin{aligned}
g_{qq}(\eta,\eta') &= g_{pp}(\eta,\eta') = g_{\mathcal{H}\mathcal{H}}(\eta,\eta') = \Theta(\eta-\eta')\,, \\
g_{qp}(\eta,\eta') &= -2\left(e^{-\eta/2} - e^{-\eta'/2}\right)\Theta(\eta-\eta')\,, \\
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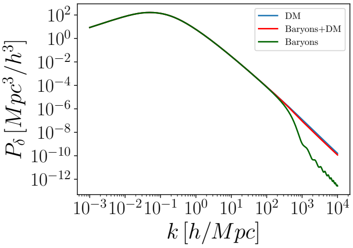
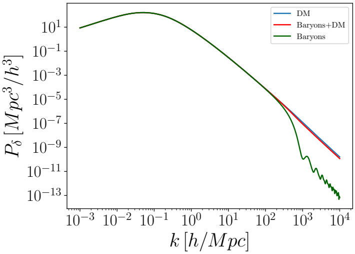
and potentials

$$
\begin{aligned}
v^b_{P_p}(k) &= \frac{\gamma-1}{\gamma\bar{\rho}^b_m}\frac{a_i e^{3\eta/2}}{H_0^2}w(k)\,,\\
v^b_{P_{\mathcal{H}}}(k) &= \frac{\gamma-1}{\bar{\rho}^b}e^{-\eta/2}w(k)\,,\\
v^{\alpha\gamma}_G(k) &= -\frac{3\Omega^\alpha_{m,0}}{2k^2\bar{\rho}^\alpha}e^{\eta/2}\,.
\end{aligned}
$$

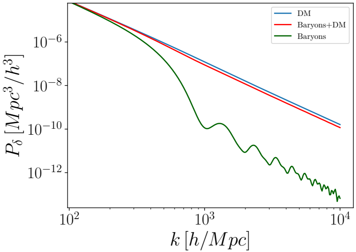
Matter Power Spectrum in Expanding Spacetime



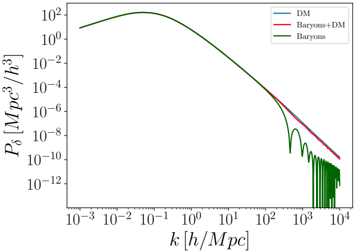
(j) no heating



(k) $s = -26.5$

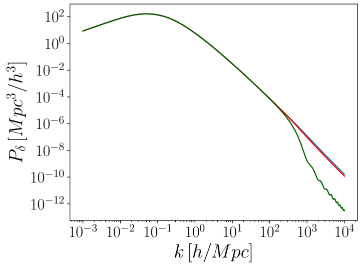


(l) $s = -26.5$, zoom in

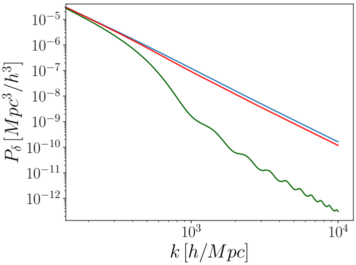


(m) $s = -25.5$

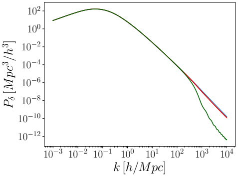
Matter Power Spectrum (Not the Cosmological Model)



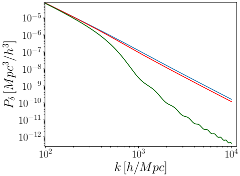
(n) $s = -24$



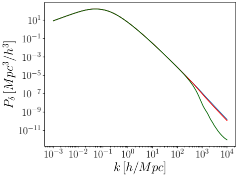
(o) $s = -24$, zoom in



(p) $s = -23.5$



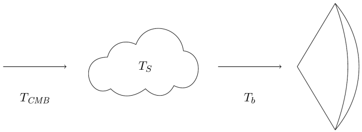
(q) $s = -23.5$, zoom in



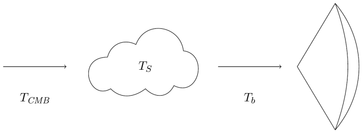
(r) $s = -23$

Outline

Differential Brightness Temperature

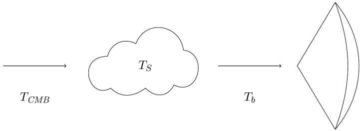


Differential Brightness Temperature



$$\delta T_b(z) = T_b - T_{CMB} = 9\,\mathrm{mK}(1-x)(1+\delta)(1+z)^{1/2}\,\frac{T_S - T_{CMB}}{T_S}\,.$$

Differential Brightness Temperature



Assume $T_S \approx T_{CMB}$:

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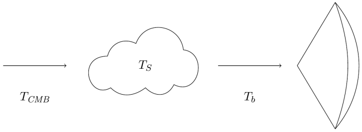
Differential Brightness Temperature



Assume $T_S \approx T_{CMB}$:

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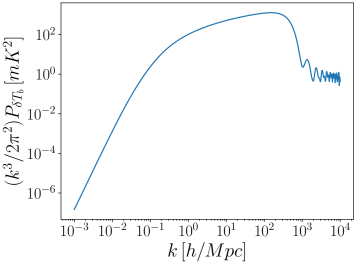
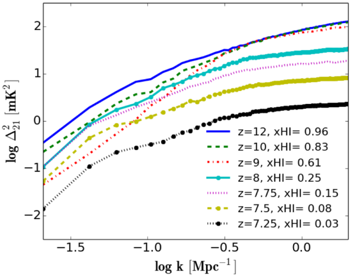
Compute the power spectrum

$$\begin{aligned}\langle \delta T_b(1)\delta T_b(2)\rangle &= (9\mathrm{mK})^2\,\frac{(1+z)}{\bar{\rho}^2}\langle (x_{HI}\,\rho)(1)(x_{HI}\,\rho)(2)\rangle\\ &= (9\mathrm{mK})^2\,\frac{(1+z)}{\bar{\rho}^2}\left\{\left[\frac{1}{2}+x_{T_0}\,T_0\right]^2\langle\rho(1)\rho(2)\rangle\right.\\ &\left.-2\left[\frac{1}{2}+x_{T_0}\,T_0\right]\frac{2x_{T_0}}{5k_B}\langle\rho(1)h(2)\rangle+\left(\frac{2x_{T_0}}{5k_B}\right)^2\langle h(1)h(2)\rangle\right\}\,.\end{aligned}$$

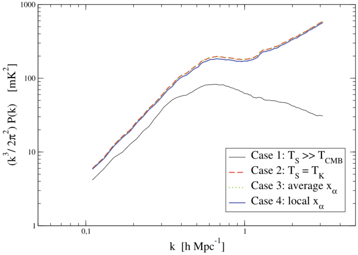
21cm Power Spectrum



(s) RKFT result



(t) Hassan et al. 2016



(u) Baek et al. 2009

The Problem With the Adiabatic Cooling

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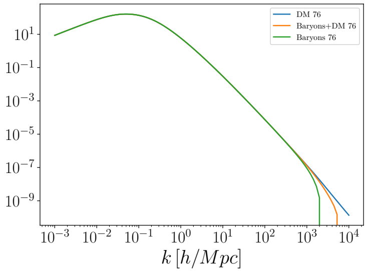
\end{aligned}
$$

This results in the enthalpy propagator

$$
g_{\mathcal{H}\mathcal{H}}(\eta_1,\eta_2) = e^{3(\gamma-1)(\eta_2-\eta_1)} = (a_2/a_1)^2 .
$$

The Problem With the Adiabatic Cooling

Computing the power spectrum with the enthalpy propagator $g_{\mathcal{H}\mathcal{H}}(\eta_1, \eta_2) = (a_2/a_1)^2$ and without heating:



The Problem With the Mean Ionisation Fraction

Compute the mean ionisation fraction via

$$\langle x \rangle = \frac{1}{2} - x_{T_0} \left[T_0 - \frac{2}{5Nk_B} \langle \mathcal{H} \rangle \right] .$$

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Therefore we need to compute

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$$\cong \sum_{a=b,d} \sum_{\alpha=0,1,2} \int \mathrm{d}2\, \Delta_{f_1^b \beta_\alpha^a}(1,2)\nu_{\beta_\alpha^a}(-2) .$$

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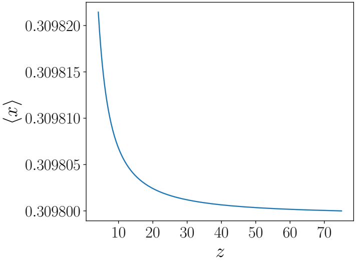

Figure: mean ionisation fraction

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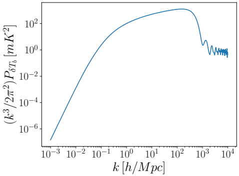
    Able to compute power spectra of ionised hydrogen (or differential brightness temperature power spectra)



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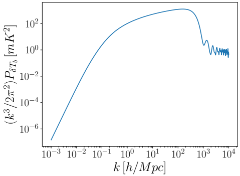

$\rightarrow$ The epoch of reionization can be modelled using RKFT and observables like the power spectra of ionised hydrogen can be easily calculated!